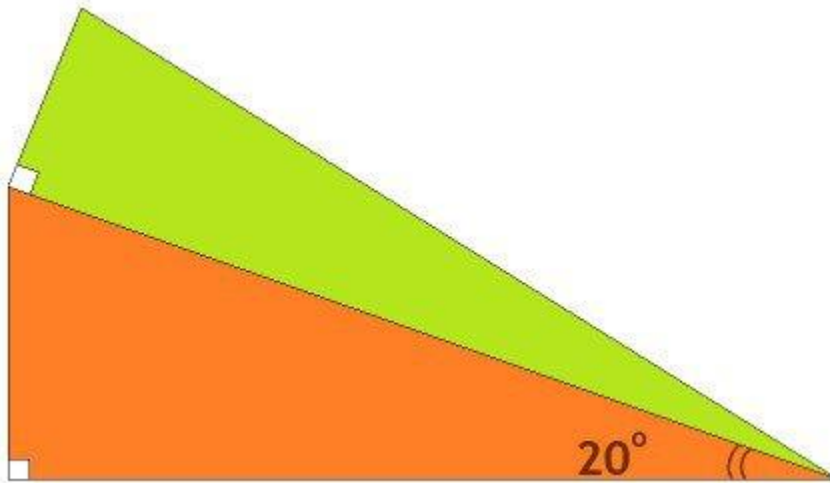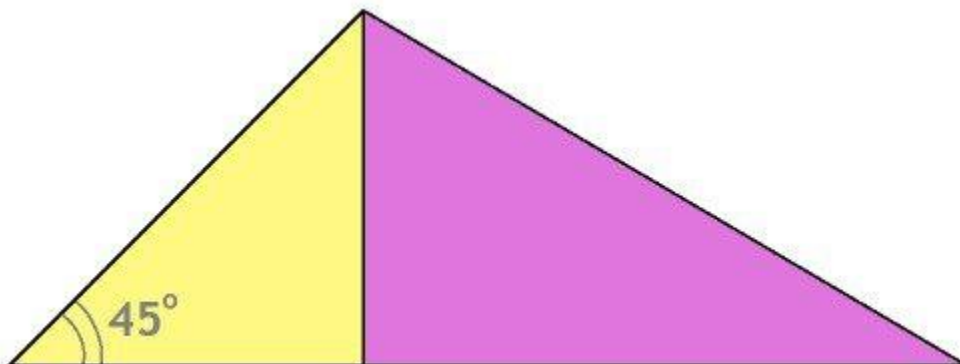# Advanced Trigonometry

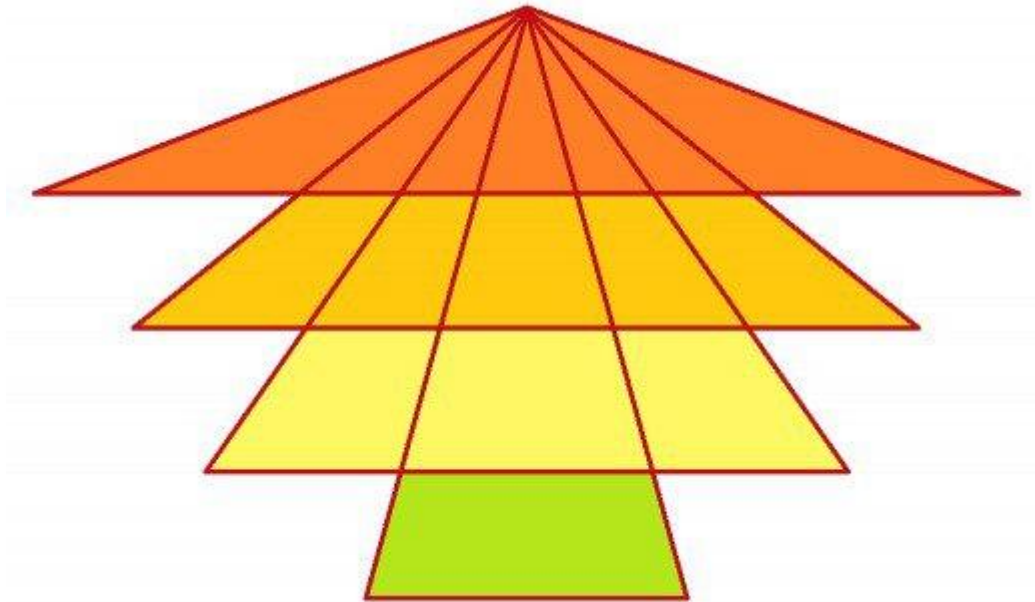## (1) Right Triangles of Equal Perimeter

Problem: The two right triangles shown above have equal perimeters. The hypotenuse of the orange triangle is one leg of the green triangle stacked on top of it. If the smallest angle of the orange triangles is 20 degrees, what are the angles of the green right triangle?

(2) Find the Missing Angles

Problem: Consider a triangle with one angle equal to 45 degrees. An altitude is drawn to partition the original triangle into two smaller right triangles, colored yellow and purple in the diagram above. If the area of the purple triangle is 1.5 times the area of the yellow triangle, what are the angles of the original whole triangle?

(3) Isosceles Integer Triangle

Problem: An isosceles triangle has a perimeter of 18 and integer side lengths. What is the smallest possible angle measure at one of its corners?

(4) Bisected Angle
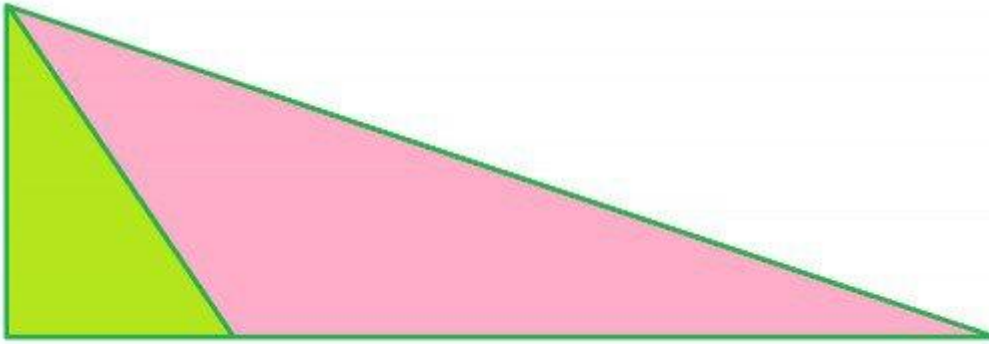
Problem: Consider a scalene right triangle such that if you draw a line that bisects the mid-sized angle, you end up with two smaller triangles, one of which has three times the area of the other. What are the angles of this triangle?

(5) Angle at the Apex of a Cone
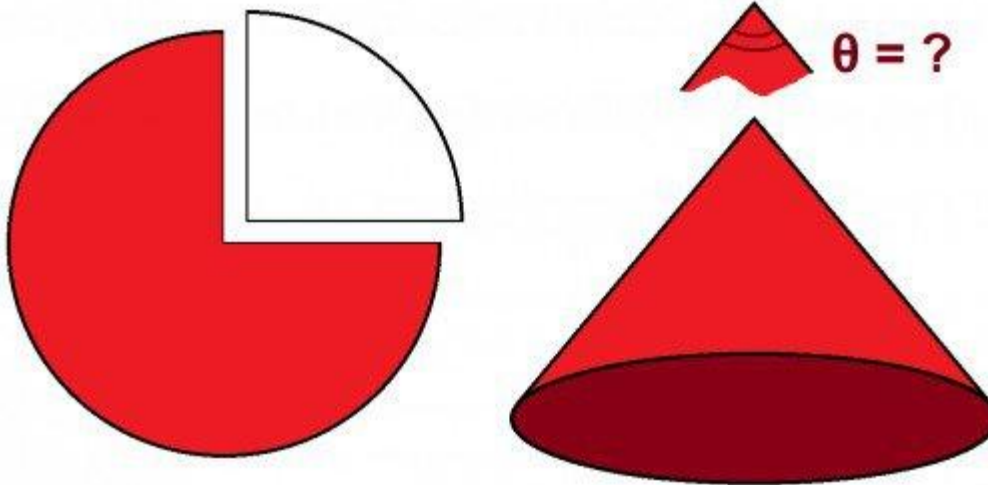
Problem: A quarter circular sector or is removed from a circle and the remainder is folded into a cone by connecting the cut edges. When viewed from the side, what is the angle at the apex of the cone?

(6) Maximum Area of a Triangle
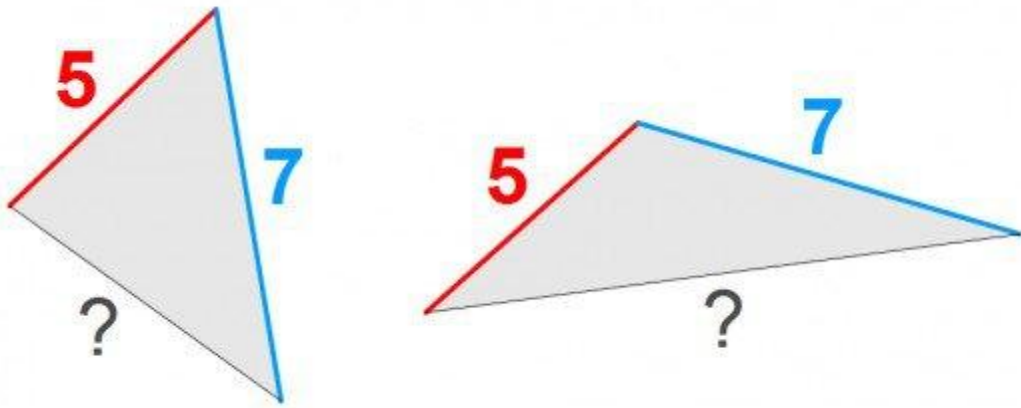
Problem: Two sides of a triangle have lengths 5 and 7. What should the length of the third side be so that the area of the triangle is maximized?
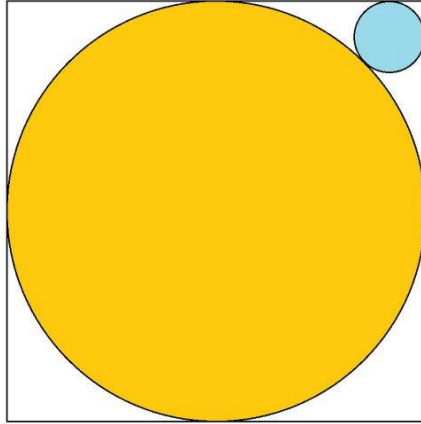
(7) Right Triangles of Equal Area
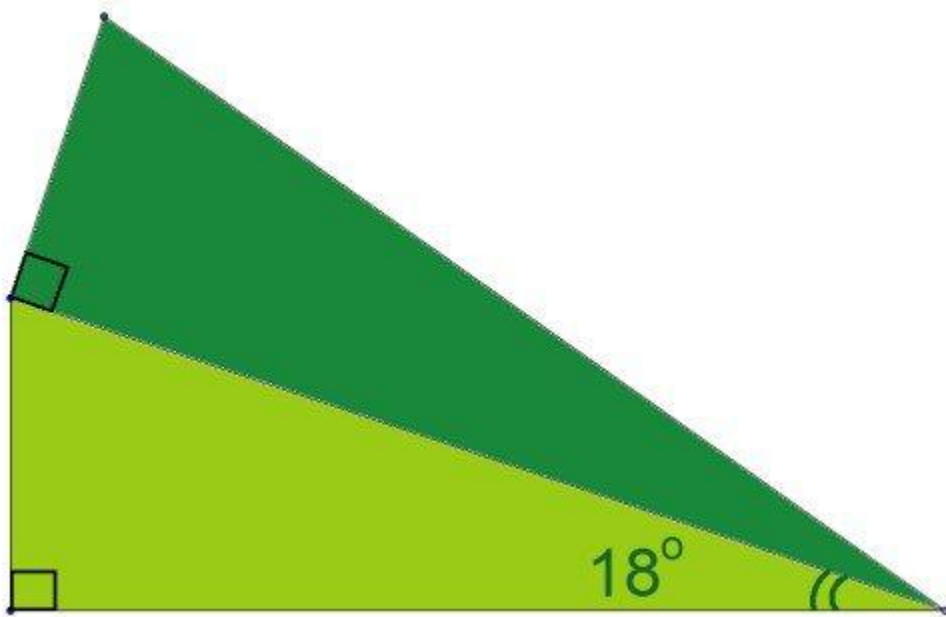
Problem: Two right triangles share a common side and have equal areas. The hypotenuse of the light green triangle is one of the legs of dark green triangle stacked on top of it. If the smallest angle of light green triangle is 18 degrees, what are the angles of the dark green triangle?
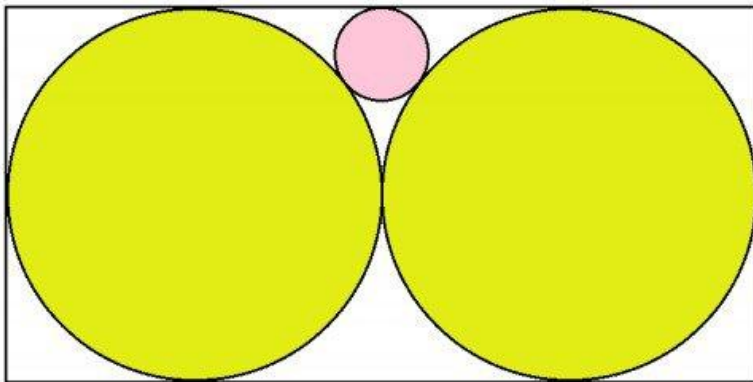
Advanced Geometry

(8)

A circle with a radius of 1 is inscribed within a 2-by-2 square. A smaller circle is inscribed in the corner such that it is tangent to two adjacent sides of the square and the larger circle. What is the radius of this smaller circle?

(9)

Two circles each with a radius of 1 and tangent to each other are inscribed in a 2-by-4 rectangle. A smaller circle is inscribed in the space between the circles and the long edge of the rectangle, such that it is tangent to both circles and the edge of the rectangle. What is the radius of this smaller circle?

(10)
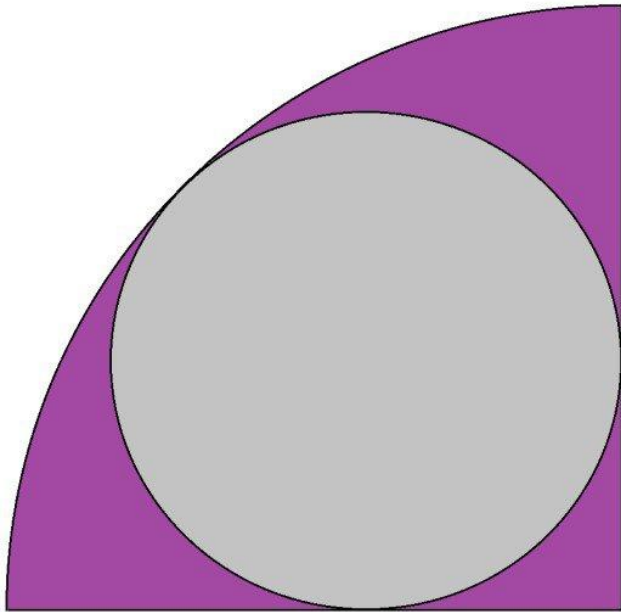
A circle is inscribed within a quarter-circle sector. It is tangent to the arc of the sector and the two perpendicular radii of the sector. What is the ratio of the area of the circle to the area of the sector ?

(11)
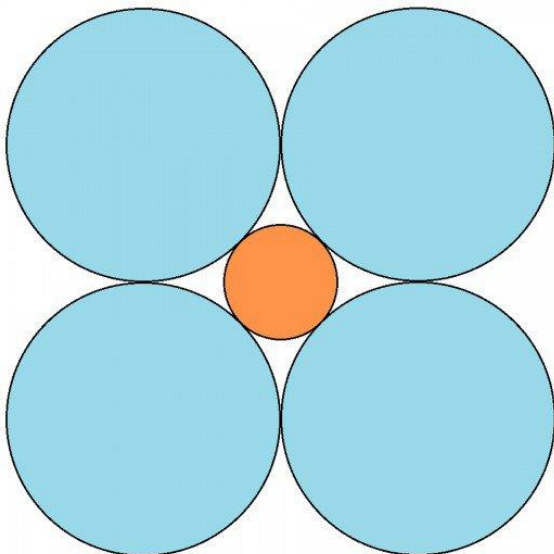
Four circles, each with a radius of 1, are arranged so that they are tangent to two others and their centers form the corners of a square. A smaller circle is inscribed in the space bounded by the four circles, and this smaller circle is tangent to each of the other four. What is its radius?
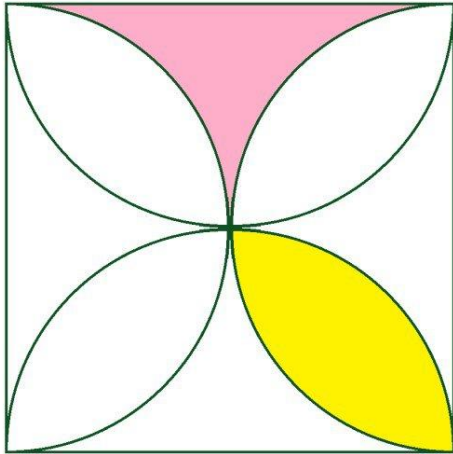
(12)
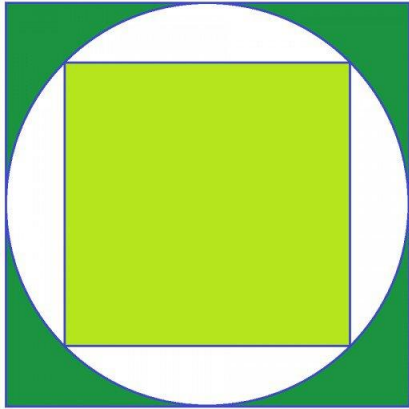
A circle is inscribed within a square. A smaller square is inscribed within this circle. What is the ratio of the area of the larger square to that of the smaller square?

(13)
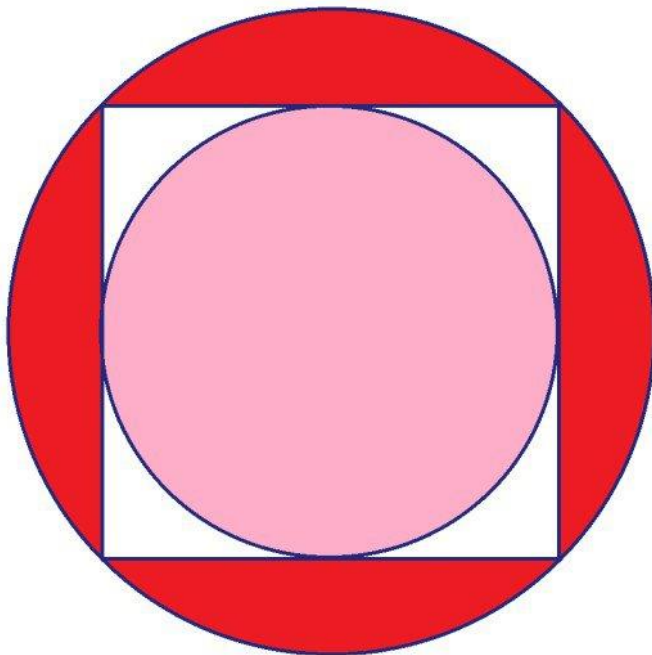
A square is inscribed in a circle. Within this square is inscribed a smaller circle. What is the ratio of the area of the larger circle to that of the smaller circle?
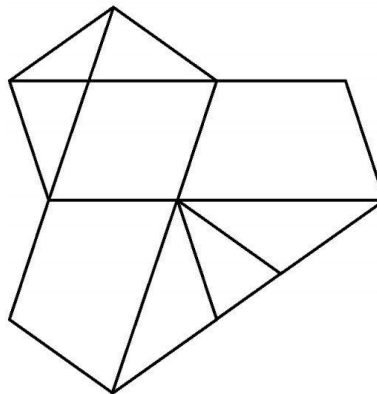
(14)

The figure above shows a square with a side length of 1. Along the edges are four half-circles with each a radius of 0.5. Their intersections inside the square create eight regions: four like the one shown in pink, and four like the one shown in yellow. What are the areas of the pink and yellow regions?
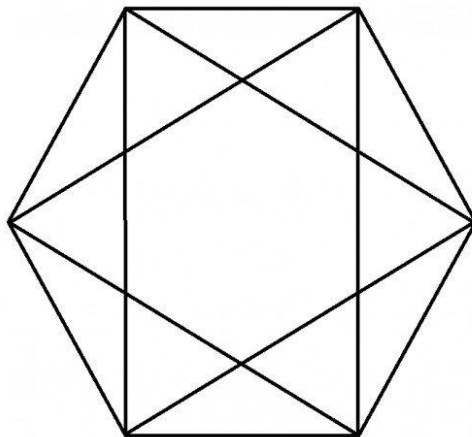
Geometry Puzzles

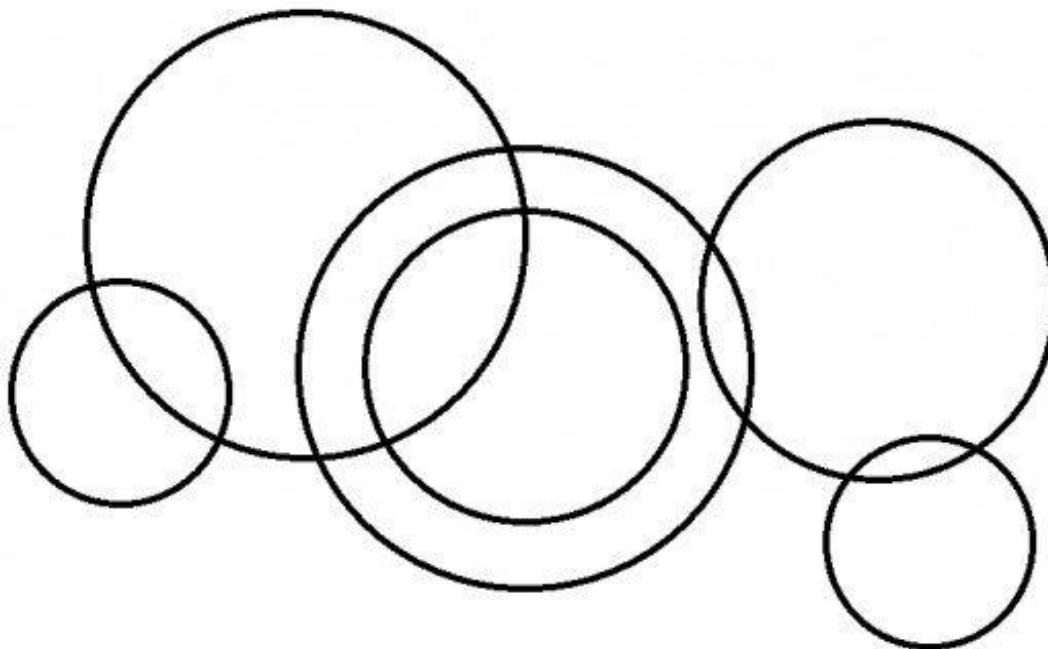## (15) How many pentagons are in the figure below?

(16) How many rectangles are in the figure below?
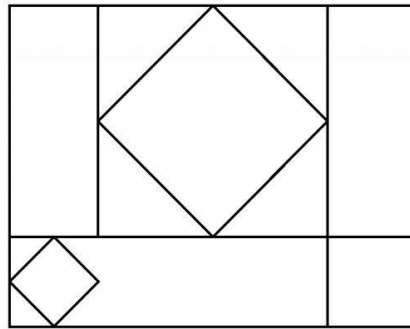
**(17) How many triangles are in the figure below?**

(18) How many circular arcs are in the figure below?
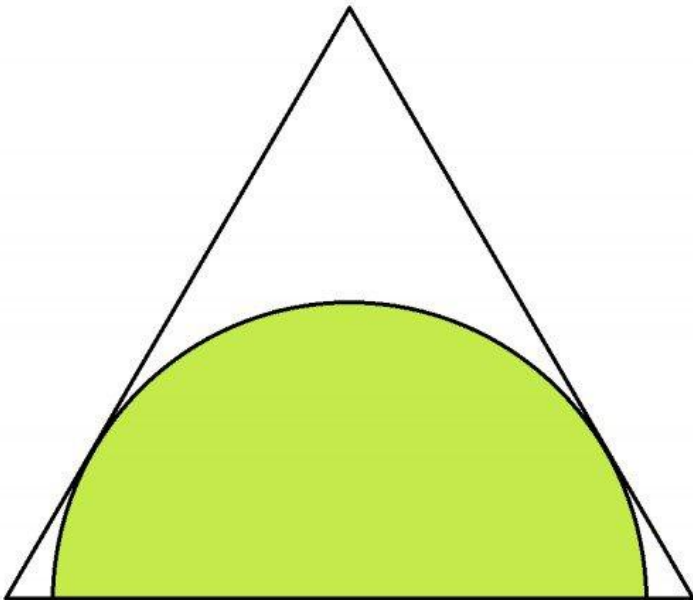
Inscribed Circles and Equilateral Triangles

Geometry problems involving circles inscribed in equilateral triangles, and equilateral triangles inscribed in circles are often found in brain teasers and on standardized tests such as the SAT, GMAT, and GED. To determine the absolute or relative sizes of the triangles and circles, you only need to a few basic geometric properties of these shapes.

One property is that if a line L is tangent to a circle at point X, then the line segment connecting X and the center of the circle is perpendicular to L. Also, if you cut an equilateral triangle in half along its altitude, the resulting half-triangles are 30°-60°-90° triangles. The ratio of the side lengths of a 30°-60°-90° triangle are 1: sqrt (3): 2. With these essential geometric facts and a few other properties you can solve any inscribed shape problem.

 (19)
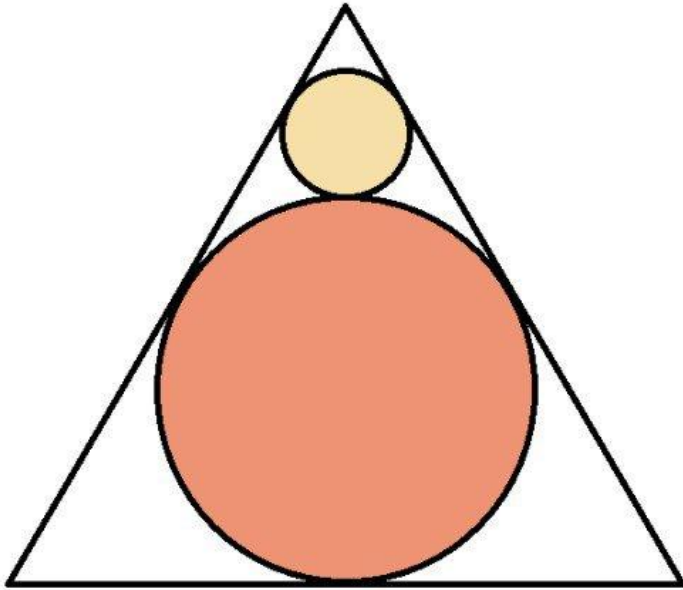
A circle is inscribed within an equilateral triangle. A smaller circle is inscribed in the space between the circle and two edges of the equilateral triangle. If the triangle has an edge length of 1, what are the radii of the large and small circles?

(20)

A half-circle is inscribed within an equilateral triangle such that the diameter of the half-circle is centred on one edge of the triangle and the arc is tangent to the other two sides. What is the diameter of the semi-circle if the triangle has an edge length of 4?
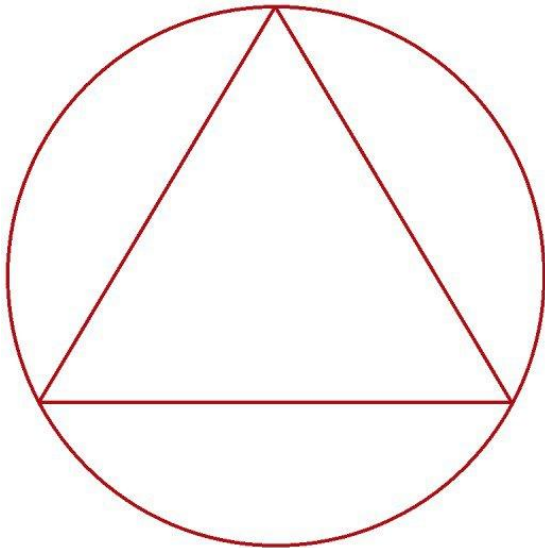
(21)
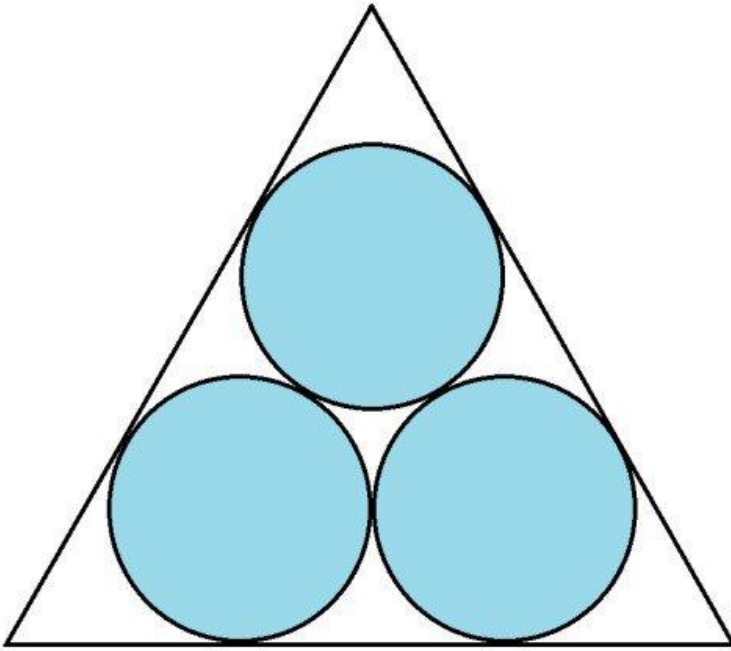
Three circles each with a radius of 1 are inscribed within an equilateral triangle such that the three circles are tangent to each other and to two edges of the triangle. What is the side length of the triangle?
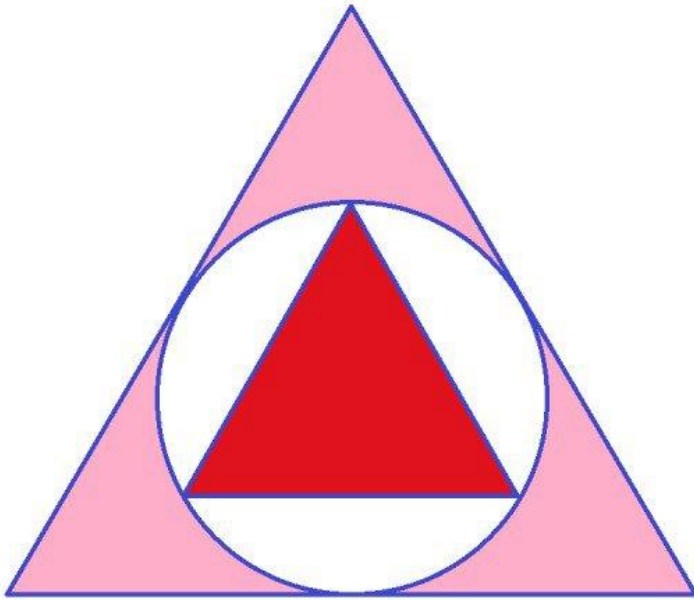
(22)
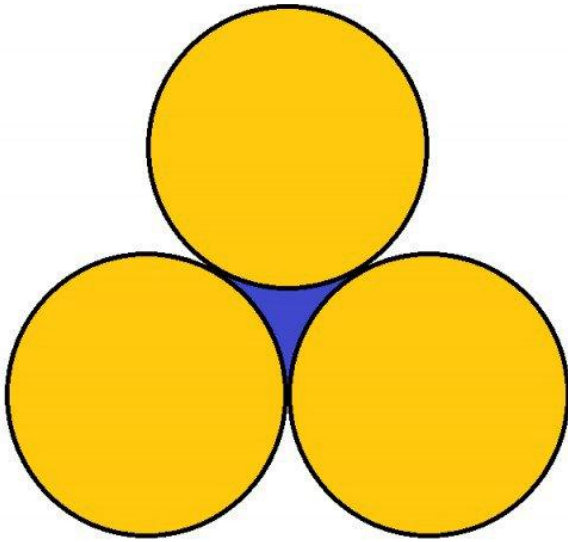
Three circles, each with a radius of 2, are mutually tangent. What is the area of the region bounded by the three circles, shown in blue above?
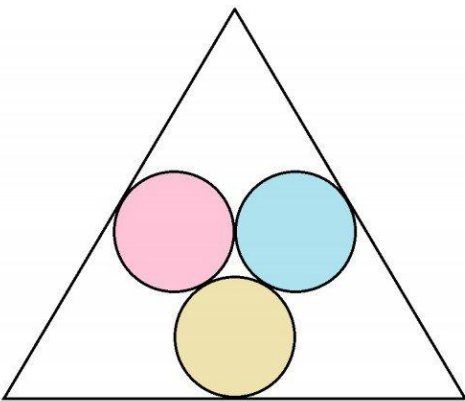
(23)

An equilateral triangle is inscribed within a circle whose radius is sqrt(3). What is the area of the triangle?

(24)

A circle is inscribed within an equilateral triangle. An equilateral triangle is inscribed within this circle. What is the ratio of the area of the larger triangle to the smaller triangle?

(25)

Three circles are inscribed within an equilateral triangle such that they are tangent to one another and to the midpoint of one edge of the triangle. If each circle has a radius of 1, what is the side length of the triangle?

## (26) BONUS Problem: Cut of Minimal Length That Splits Triangle into 2 Equal Pieces
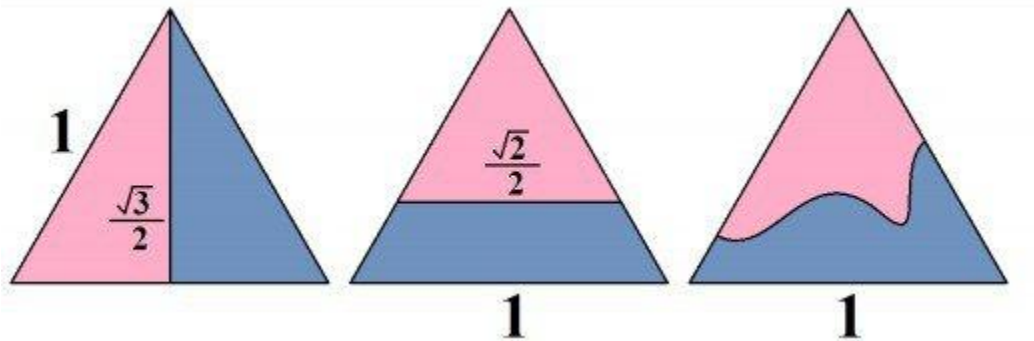
Consider an equilateral triangle whose side length is 1. What is the shape and minimal length of a curve or line that will partition the triangle into two pieces of equal area? Some example lines and arcs are shown above.

(27) BONUS Problem
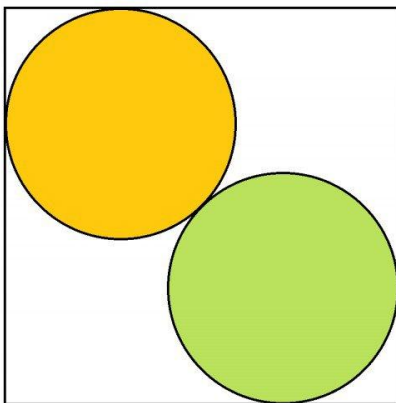
Two circles of equal size are inscribed in a square such that the circles are tangent to each other and to two sides of the square. The centers of the circle lie on the square's diagonal. If the radius of each circle is 1, what is the side length of the square?

(28) BONUS Problem
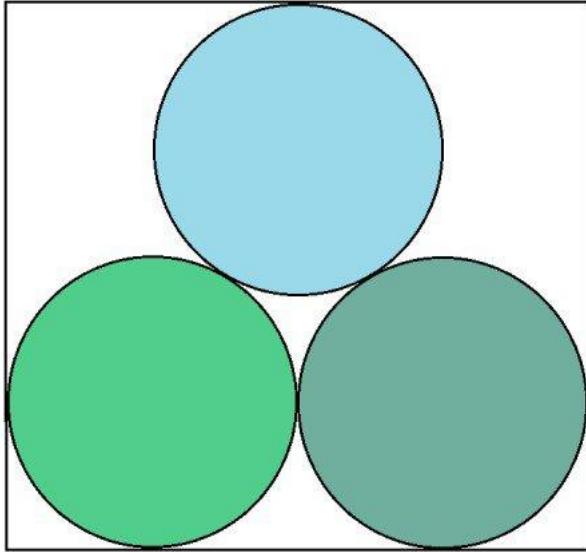
Three circles have a radius of 6 and are inscribed in a rectangle as shown in the figure above. What is the area and perimeter of the rectangle?